Distinguishing quantum channels via magic squares game

M. Ramzan and M. K. Khan

Department of Physics Quaid-i-Azam University

Islamabad 45320, Pakistan

(Dated: January 2, 2010)

We study the effect of quantum memory in magic squares game when played in quantum domain. We consider different noisy quantum channels and analyze their influence on the magic squares quantum pseudo-telepathy game. We show that the probability of success can be used to distinguish the quantum channels. It is seen that the mean success probability decreases with increase of quantum noise. Where as the mean success probability increases with increase of quantum memory. It is also seen that the behaviour of amplitude damping and phase damping channels is similar. On the other hand, the behaviour of depolarizing channel is similar to the flipping channels. Therefore, the probability of success of the game can be used to distinguish the quantum channels.

Keywords: Quantum magic squares game; quantum memory; success probability

I. INTRODUCTION

Applications of game theory [1] can be found in several research areas, such as economics, biology, physics and computer sciences. In the recent past, rapid interest has been developed in the discipline of quantum information [2] that has led to the creation of quantum game theory [3]. During last few years, number of authors have contributed to the development of quantum game theory [4-9]. In this direction, much work has been done on quantum prisoners' dilemma game [10-12] and other games have been converted to the quantum realm including the battle of the sexes [8, 13], the Monty Hall problem [14, 15], the rock-scissors-paper [16], and the others [17-20]. Quantum pseudo-telepathy game [21] is something which can not be won in the classical world without communication but can be won in the quantum world using entangled state without any use of classical communication. Brassard et al. [21] have shown that how to win the magic

square game for $n = 3$ with certainty by sharing a two qubit entanglement between Alice and Bob. Gawron et al. [19] have studied noise effects in quantum magic squares game. They have shown that the probability of success can be used to determine the characteristics of quantum channels. Recently, Sousa et al. [22] have proposed a quantum game which can be used to control the access of processes to the CPU in a quantum computer. More recently, James et al. [23] have analyzed the quantum penny flip game using geometric algebra.

Decoherence is an integral part of the theory of quantum computation and communication. A major problem of quantum communication is to faithfully transmit unknown quantum states through a noisy quantum channel. When quantum information is sent through a channel (optical fiber), the carriers of the information interact with the channel and get entangled with its many degrees of freedom. This gives rise to the phenomenon of decoherence on the state space of the information carriers. To deal with the problem of decoherence, two methods have been developed, known as quantum error correction [24] and entanglement purification [25]. When quantum information is processed in the real-world, the decoherence caused by the external environment is inevitable. Implementation of decoherence in quantum games have been studied by different authors [6, 26]. Recently, interest has been developed in implementing quantum memory in the field of quantum game theory [8].

Quantum channels are implemented by suitable quantum devices consisting of intrinsic degrees of freedom associated with the environment and acting on the system via particular interactions between the system and the environment. The assumption that noise is uncorrelated between successive uses of a channel is not realistic. Hence memory effects need to be taken into account. Quantum channels with memory [27-29] are the natural theoretical framework for the study of any noisy quantum communication system where correlation time is longer than the time between consecutive uses of the channel. A more general model of a quantum channel with memory was introduced by Bowen and Mancini [30] and also studied by Kretschmann and Werner [31].

In this paper, we study the quantum magic squares game influenced by different memory channels, such as amplitude damping, depolarizing, phase damping, bit-flip, phase-flip and bit-phase-flip channels, parameterized by a quantum noise parameter α and memory parameter μ. Here $\alpha \in [0, 1]$ and $\mu \in [0, 1]$ represent the lower and upper limits of quantum noise parameter and memory parameter respectively. It is seen that the behaviour of the damping (amplitude and phase damping) channels is different as compared to the depolarizing and flipping channels. Therefore, the quantum channels can easily be distinguished using the success probability of the game.

II. THE MAGIC SQUARES GAME

The magic squares game is a two-player game presented by Aravind [32] which was primarily built on the work done by Mermin [33]. The magic square is a 3×3 matrix filled with numbers 0 or 1, such that the sum of entries in each row is even and the sum of entries in each column is odd. However, it is impossible to have such a matrix with all the rows having even parity and all the columns having odd parity. Therefore, there can be no classical strategy that always wins. In the magic squares game, there are two players, say, Alice and Bob. Alice is given the number of the row and Bob is given the number of the column. Alice gives the entries for a row and Bob gives entries for a column. The winning condition is that the parity of the row must be even, the parity of the column must be odd, and the intersection of the given row and column must be same. During the game, Alice and Bob are not allowed to communicate with each other. There exists a (classical) strategy that leads to winning probability of 8/9. If parties are allowed to share a quantum state they can achieve probability 1. An interesting feature of this game is that it does not require a promise.

In the quantum version of this game [21], Alice and Bob share an entangled quantum state of the form

$$|\Psi_{in}\rangle = \frac{1}{2}\left(|0011\rangle - |1100\rangle - |0110\rangle + |1001\rangle\right) \tag{1}$$

In equation (1) the first two qubits correspond to Alice, whereas the last two qubits correspond to Bob. Depending upon their inputs (the specific row and column to be filled in) Alice and Bob

apply unitary operators $A_i \otimes I$ and $I \otimes B_j$, respectively,

$$
\begin{aligned}
A_1 &= \frac{1}{\sqrt{2}}\begin{bmatrix} i & 0 & 0 & 1 \\ 0 & -i & 1 & 0 \\ 0 & i & 1 & 0 \\ 1 & 0 & 0 & i \end{bmatrix}, \qquad B_1 = \frac{1}{2}\begin{bmatrix} i & -i & 1 & 1 \\ -i & -i & 1 & -1 \\ 1 & 1 & -i & i \\ -i & i & 1 & 1 \end{bmatrix} \\
A_2 &= \frac{1}{2}\begin{bmatrix} i & 1 & 1 & i \\ -i & 1 & -1 & i \\ i & 1 & -1 & -i \\ -i & 1 & 1 & -i \end{bmatrix}, \qquad B_2 = \frac{1}{2}\begin{bmatrix} -1 & i & 1 & i \\ 1 & i & 1 & -i \\ 1 & -i & 1 & i \\ -1 & -i & 1 & -i \end{bmatrix} \\
A_3 &= \frac{1}{2}\begin{bmatrix} -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix}, \qquad B_3 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}
\end{aligned} \tag{2}
$$

where i and j denote the corresponding input. Finally, Alice and Bob measure their qubits in the computational basis. Further steps to calculate the success probability of the game are given in the next section.

A. Quantum memory channels

A major hurdle in the path of efficient information transmission is the presence of noise, in both classical and quantum channels. This noise causes a distortion of the information sent through the channel. Error correcting codes are used to overcome this problem. Messages are encoded into code-words, which are then sent through the channel. Information transmission is said to be reliable if the probability of error, in decoding the output of the channel, vanishes asymptotically in the number of uses of the channel. A basic question of information theory is whether there is any advantage in using entangled states as input states. That is, whether or not encoding the classical data into entangled rather than separable states increases the mutual information. For the case when multiple uses of the channel are not correlated, there is no advantage in using entangled states. Correlated noise, also referred as memory in the literature, acts on consecutive uses of the channels. However, in general, one may want to encode classical data into entangled strings or consecutive uses of the channel may be correlated to each other. Hence, we are dealing with a strongly correlated quantum system, the correlation of which results from the memory of the

channel itself.

In ref. [26] a Pauli channel with partial memory was studied. The action of the channel on two consecutive qubits is given by the Kraus operators

$$A_{ij} = \sqrt{\alpha_i[(1-\mu)\alpha_j + \mu\delta_{ij}]}\sigma_i \otimes \sigma_j \tag{3}$$

where σ_i (σ_j) are usual Pauli matrices, α_i (α_j) represent the quantum noise and indices i and j runs from 0 to 3. The above expression means that with probability μ the channel acts on the second qubit with the same error operator as on the first qubit, and with probability $(1-\mu)$ it acts on the second qubit independently. Physically the parameter μ is determined by the relaxation time of the channel when a qubit passes through it. In order to remove correlations, one can wait until the channel has relaxed to its original state before sending the next qubit, however this lowers the rate of information transfer. Thus it is necessary to consider the performance of the channel for arbitrary values of μ to reach a compromise between various factors which determine the final rate of information transfer. Thus in passing through the channel any two consecutive qubits undergo random independent (uncorrelated) errors with probability $(1-\mu)$ and identical (correlated) errors with probability μ. This should be the case if the channel has a memory depending on its relaxation time and if we stream the qubits through it. The action of the Pauli channels on n-qubits can be generalized in Kraus operator form as given below

$$A_{i_1.....i_n} = \sqrt{\alpha_{i_n}\prod_{m=1}^{n-1}[(1-\mu)\alpha_{i_m} + \mu\delta_{i_m,i_{m+1}}]\sigma_{i_1} \otimes ..... \otimes \sigma_{i_n}} \tag{4}$$

As stated above, with probability $(1-\mu)$ the noise is uncorrelated and can be completely specified by the Kraus operators

$$D^u_{ij} = \sqrt{\alpha_i\alpha_j}\sigma_i \otimes \sigma_j, \tag{5}$$

and with probability μ the noise is correlated (i.e. the channel has memory) which can be specified by the Kraus operators

$$D^c_{kk} = \sqrt{\alpha_k}\sigma_k \otimes \sigma_k, \tag{6}$$

A detailed list of single qubit Kraus operators for different quantum channels with uncorrelated noise is given in table 1. The action of such a channel if n qubits are streamed through it, can be described in operator sum representation as [2]

$$\rho_f = \sum_{k_1,....,.k_n=0}^{n-1}(A_{k_n} \otimes .....A_{k_1})\rho_{in}(A^\dagger_{k_1} \otimes .....A^\dagger_{k_n}) \tag{7}$$

where ρ_{in} represents the initial density matrix for quantum state and A_{k_n} are the Kraus operators expressed in equation (4). The Kraus operators satisfy the completeness relation

$$\sum_{k_n=0}^{n-1} A_{k_n}^{\dagger} A_{k_n} = 1 \tag{8}$$

However, the Kraus operators for a quantum amplitude damping channel with correlated noise are given by Yeo and Skeen [27] as given as

$$A_{00}^{c} = \begin{bmatrix} \cos\chi & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_{11}^{c} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \sin\chi & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

where, $0 \leq \chi \leq \pi/2$ and is related to the quantum noise parameter as

$$\sin\chi = \sqrt{\alpha} \tag{10}$$

It is clear that A_{00}^{c} cannot be written as a tensor product of two two-by-two matrices. This gives rise to the typical spooky action of the channel: $|01\rangle$ and $|10\rangle$, and any linear combination of them, and $|11\rangle$ will go through the channel undisturbed, but not $|00\rangle$ .The action of this non-unital channel is given by

$$\pi \to \rho = \Phi(\pi) = (1-\mu) \sum_{i,j=0}^{1} A_{ij}^{u} \pi A_{ij}^{u\dagger} + \mu \sum_{k=0}^{1} A_{kk}^{c} \pi A_{kk}^{c\dagger} \tag{11}$$

The action of the super-operators provides a way of describing the evolution of quantum states in a noisy environment. In our scheme, the Kraus operators are of the dimension 2^3. They are constructed from single qubit Kraus operators by taking their tensor product over all n^3 combinations

$$A_k = \bigotimes_{k_n} A_{k_n} \tag{12}$$

where n is the number of Kraus operator for a single qubit channel. The final state of the game after the action of the channel can be obtained as

$$\rho_f = \Phi_{\alpha,\mu}(|\Psi\rangle\langle\Psi|) \tag{13}$$

where $\Phi_{\alpha,\mu}$ is the super-operator realizing the quantum channel parametrized by real numbers α and μ. After the action of the players unitary operations, the game's final state transforms to

$$\rho_{\acute{f}} = (A_i \otimes B_j)(\Phi_{\alpha,\mu}(|\Psi\rangle\langle\Psi|)(A_i^{\dagger} \otimes B_j{}^{\dagger}) \tag{14}$$

The probability of success $P_{i,j}(\alpha,\mu)$ can be computed as the probability of measuring $\rho_{\acute{f}}$ in the state indicating success

$$P_{i,j}(\alpha,\mu) = \mathrm{Tr}(\rho_{\acute{f}} \sum_{m} |\xi_m\rangle \langle\xi_m|) \tag{15}$$

where $|\xi_m\rangle$ are the states that imply success and Tr represents the trace of the matrix. The mean probability of success of the game is calculated from

$$\bar{P}_{i,j}(\alpha,\mu) = \sum_{i,j\in\{1,2,3\}} P_{i,j}(\alpha,\mu). \tag{16}$$

It can be easily checked that the results of ref. [19] can be reproduced if we put $\mu = 0$ in tables 2 and 3 for success probability and mean success probability respectively.

III. RESULTS AND DISCUSSIONS

We compute the success probability $P_{i,j}(\alpha,\mu)$ for different inputs ($i, j \in \{1,2,3\}$) in the magic squares game for different quantum channels such as depolarizing, amplitude damping, phase damping and flipping channels. It is seen that the mean probability of success, $\bar{P}_{i,j}(\alpha,\mu)$, heavily depends on the quantum noise parameter α and memory parameter μ (see table 3). The game results for each combination of operators A_i, B_j for depolarizing, amplitude damping, phase damping, bit flip, phase flip and bit-phase flip channels are listed in table 2. In table 3, we present the results for the mean success probability $\bar{P}_{i,j}(\alpha,\mu)$. It is seen that in the case of depolarizing channel, the success probability as the function of quantum noise parameter α and memory parameter μ is the same for all the possible inputs. For amplitude damping and phase damping channels, one can observe three different types of functions. These functions are increasing for these channels under the effect of memory. The depolarizing function reaches its minimum for $\alpha = \mu = 1/2$ and is symmetrical. It is seen that in case of input $(1,3)$ the phase-flip channel does not influence the probability of success. The same is true for input $(2,3)$ for bit flip channel and also for input $(3,3)$ for bit-phase flip channel. Hence, it is possible to distinguish these channels by looking at the success probability of magic-squares game.

In figures 1 and 2, we plot mean success probability $\bar{P}(\alpha,\mu)$ as a function of quantum noise parameter α for memory parameter $\mu = 0.5$ and $\mu = 1$ respectively for amplitude damping, depolarizing, phase damping and flipping channels. It is seen that the amplitude damping, phase damping channels cause monotonic decrease of mean success probability as a function of quantum noise parameter for $\mu < 1$ (see figure 1). On the other hand, depolarizing and flipping channels give

symmetrical function. However, for $\mu = 1$, the amplitude damping and phase damping channels cause monotonic decrease of mean success probability. Where as in case of depolarizing and flipping channels, the success functions attain their maximum probability of success (see figure 2). In figures 3 and 4, we plot mean success probability $\bar{P}(\alpha, \mu)$ as a function of memory parameter μ for $\alpha = 0.5$ and $\alpha = 1$ respectively, for amplitude damping, depolarizing, phase damping and flipping channels. It is seen that the mean success probability increases linearly as a function of memory parameter μ, for $\alpha < 1$ (see figure 3). However, for depolarizing and flipping channels it reaches its maximum at $\alpha = 1$ for the entire range of the memory parameter μ.

In figures 5-8, we present the 3D graphs of mean success probability as a function of α and μ for amplitude damping, depolarizing, phase damping and flipping channels respectively. One can easily see that the mean success probability is dependent on the quantum noise parameter α and memory parameter μ. It is seen that the mean success probability decreases with α and increases with μ, which indicates that both the parameters influence the probability differently. It is also seen that the behaviour of amplitude damping and phase damping channels is same. On the other hand the behaviour of depolarizing channel is similar to the flipping channels. Therefore, it is possible to distinguish these channels by looking at the success probability of the game.

IV. CONCLUSIONS

We study quantum magic squares game under the influence of quantum memory. We analyze that how the quantum noise and memory of quantum channels can influence the magic squares quantum pseudo-telepathy game. It is seen that the mean success probability decreases with quantum noise parameter α, for $\mu < 1$. Where as it increases with μ for damping channels for $\alpha < 1$. On the other hand, for depolarizing and flipping channels the success functions attain their maximum probability of success for entire range of μ. It is further seen that the behaviour of depolarizing and flipping channels is rather different from the damping channels (amplitude and phase damping). Therefore, we can conclude with the comment that, it is possible to distinguish these channels by looking at the probability of success of the game. In other words, the probability of success can be used to distinguish the quantum channels.

[1] Rasmusen, E.: Games and Information, Basil Blackwell, Cambridge, MA (1989)

[2] Nielson, M. A., Chuang, I. L.: Quantum Computation and Quantum Information, Cambridge, Cambridge University Press (2000)

[3] Meyer, D.A.: Phys. Rev. Lett., **82,** 1052 (1999)

[4] Eisert, J., Wilkens, M., Lewenstein, M.: Phy. Rev. Lett., **83,** 3077 (1999)

[5] Marinatto, L., Weber, T.: Phys. Lett. A, **272,** 291 (2000)

[6] Flitney, A.P., Abbott, D.: J. Phys. A, **38,** 449 (2005)

[7] Cheon, T., Iqbal, A.: J. Phys. Soc. Japan, **77,** 024801 (2008)

[8] Ramzan, M., Nawaz, A., Toor, A.H., Khan, M.K.: J. Phys. A: Math. Theor., **41,** 055307 (2008)

[9] Iqbal, A., Cheon, T., Abbott, D.: Phys. Lett. A, **372,** 6564 (2008)

[10] Eisert, J., Wilkens, M.: J. Mod. Opt., **47,** 2543 (2000)

[11] Ramzan, M., Khan, M.K.: J. Phys. A: Math. Theor., **41,** 435302 (2008)

[12] Ramzan, M., Khan, M.K.: J. Phys. A: Math. Theor., **42,** 025301 (2009)

[13] Iqbal, A., Toor, A.H.: Phys. Lett. A, **280,** 249 (2001)

[14] Flitney, A.P., Abbott, D.: Phys. Rev. A, **65**, 062318 (2002)

[15] D'Ariano, G.M., Gill, R.D., Keyl, M., Kuemmerer, B., Maassen, H., Werner, R.F.: Quant. Inf. Comp., **2,** 355 (2002)

[16] Iqbal, A., Toor, A.H.: Phys. Rev. A, **65,** 022036 (2002)

[17] Iqbal, A., Toor, A.H.: Phys. Lett. A, **293,** 103 (2002)

[18] Johnson, N.F.: Phys. Rev. A, **63,** 020302(R) (2001)

[19] Gawron, P., Sladkowski, J.: Int. J. Quant. Info., **6,** 667 (2008)

[20] Lee, C.F., Johnson, N.: Phys. Lett. A **301**, 343 (2002)

[21] Brassard, G., Broadbent, A., Tapp, A.: Found. Phys. **35,** 1877 (2005)

[22] de Sousa, P.B.M., Ramos, R.V.: Quant. Inform. Process., **7**, 125 (2008)

[23] James, M., Chappell, Iqbal, A., Lohe, M.A., von Smekal, L.: quant-ph/0902.4296 (2009)

[24] Steane, A.: Phys. Rev. Lett., **77**, 793 (1996)

[25] Deutsch, D., Ekert, A., Josza, R., Macchiavello, C., Popescu, S., Sanpera, A.: Phys. Rev. Lett., **77**, 2818 (1996)

[26] Chen, L.K., Ang, H., Kiang, D., Kwek, L.C., Lo, C.F.: Phys. Lett. A, **316,** 317 (2003)

[27] Macchiavello, C., Palma, G.M.: Phys. Rev. A, **65,** 050301 (2002)

[28] Yeo Ye, Skeen Andrew, Phys. Rev. A, **67**, 064301 (2003)

[29] Karimipour, V., et. al., Phys. Rev. A, **74**, 062311 (2006)

[30] Bowen, G., Mancini, S.: Phys. Rev. A, **69**, 01236 (2004)

[31] Kretschmann, D., Werner, R.F.: Phys. Rev. A, **72**, 062323 (2005)

[32] Aravind, P.K.: Am. J. Phys. **72**, 1303 (2004)

[33] Mermin, N.D.:, Phys. Rev. Lett. **65**, 3373 (1990)

Figures captions

Figure 1. Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of quantum noise parameter α for memory parameter $\mu = 0.5$ for amplitude damping, depolarizing, phase damping and flipping (phase flip, bit flip or bit-phase flip) channels.

Figure 2. Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of quantum noise parameter α for memory parameter $\mu = 1$ for amplitude damping, depolarizing, phase damping and flipping channels.

Figure 3. Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of memory parameter μ for quantum noise parameter $\alpha = 0.5$ for amplitude damping, depolarizing, phase damping and flipping channels.

Figure 4. Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of memory parameter μ for quantum noise parameter $\alpha = 1$ for amplitude damping, depolarizing, phase damping and flipping channels.

Figure 5. Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of memory parameter μ and quantum noise parameter α for amplitude damping channel.

Figure 6. Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of memory parameter μ and quantum noise parameter α for depolarizing channel.

Figure 7. Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of memory parameter μ and quantum noise parameter α for phase flip channel.

Figure 8. Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of memory parameter μ and quantum noise parameter α for phase damping channel.

Tables Captions

Table 1. Single qubit Kraus operators for typical noise channels such as depolarizing, amplitude damping, phase damping, phase flip, bit flip and bit-phase flip where α represents the quantum noise parameter.

Table 2. Success probability $P(\alpha, \mu)$ for all combinations of magic squares game inputs for depolarizing, amplitude damping, phase damping, phase flip, bit flip and bit-phase flip channels in the presence of memory.

Table 3. Mean success probability $\bar{P}(\alpha, \mu)$ for depolarizing, amplitude damping, phase damping and flipping (phase flip, bit flip or bit-phase flip) channels in the presence of memory.

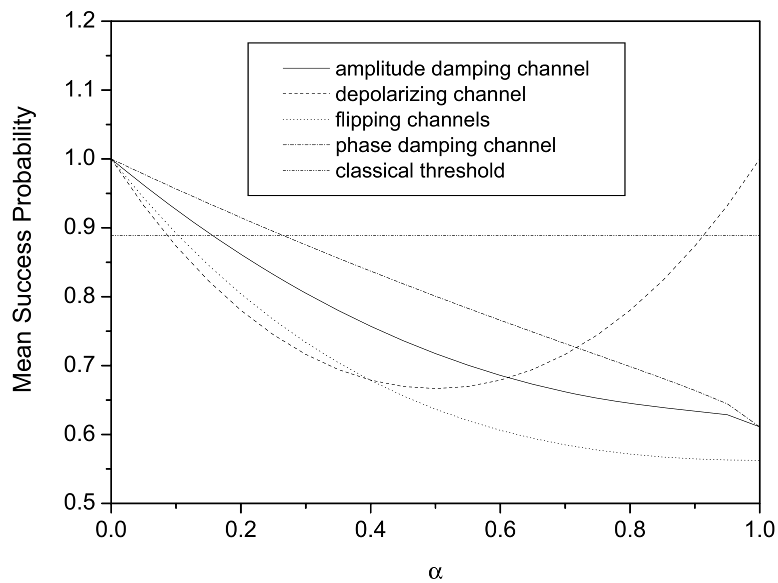

FIG. 1: Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of quantum noise parameter α for memory parameter $\mu = 0.5$ for amplitude damping, depolarizing, phase damping and flipping (phase flip, bit flip or bit-phase flip) channels.

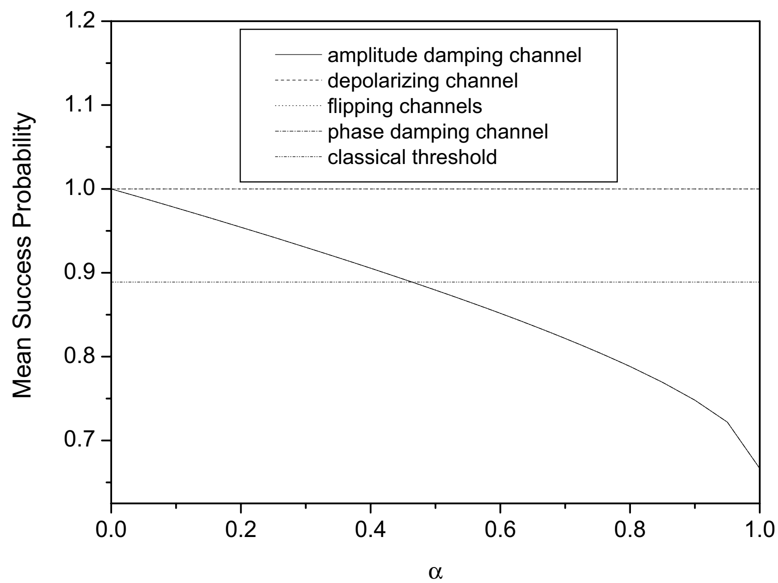

FIG. 2: Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of quantum noise parameter α for memory parameter $\mu = 1$ for amplitude damping, depolarizing, phase damping and flipping channels.

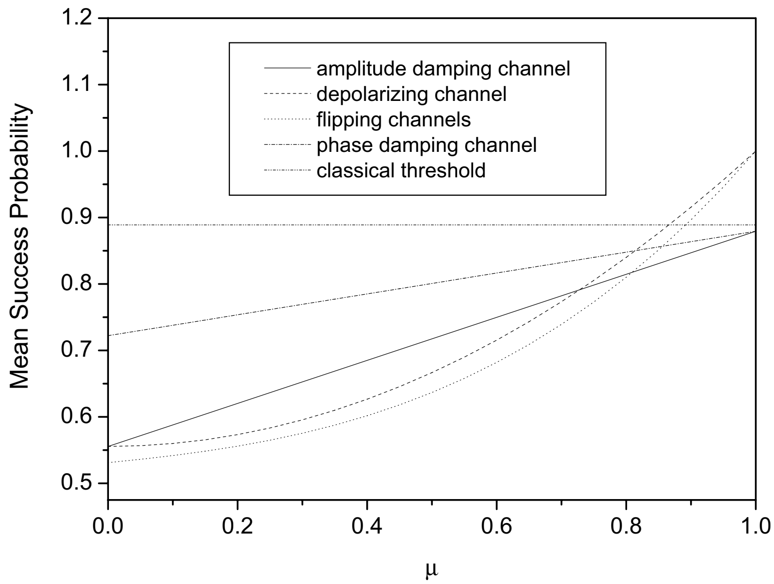

FIG. 3: Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of memory parameter μ for quantum noise parameter $\alpha = 0.5$ for amplitude damping, depolarizing, phase damping and flipping channels.

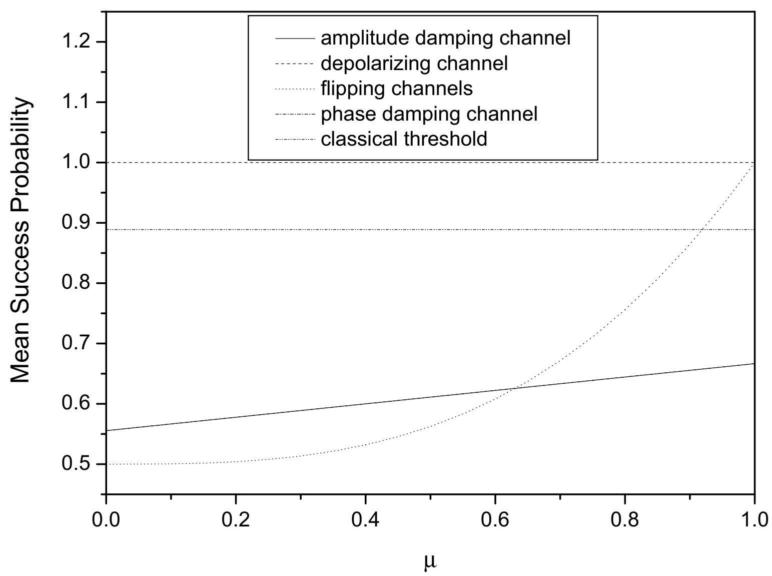

FIG. 4: Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of memory parameter μ for quantum noise parameter $\alpha = 1$ for amplitude damping, depolarizing, phase damping and flipping channels.

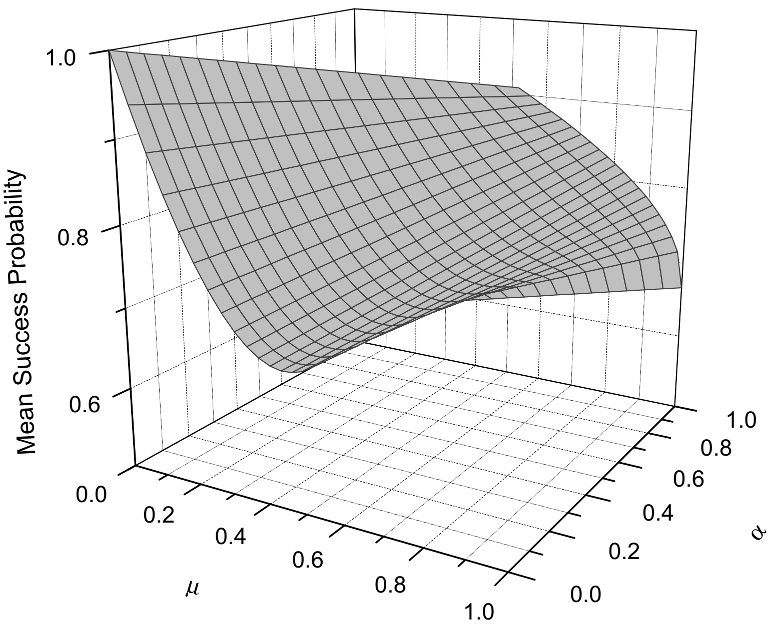

FIG. 5: Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of memory parameter μ and quantum noise parameter α for amplitude damping channel.

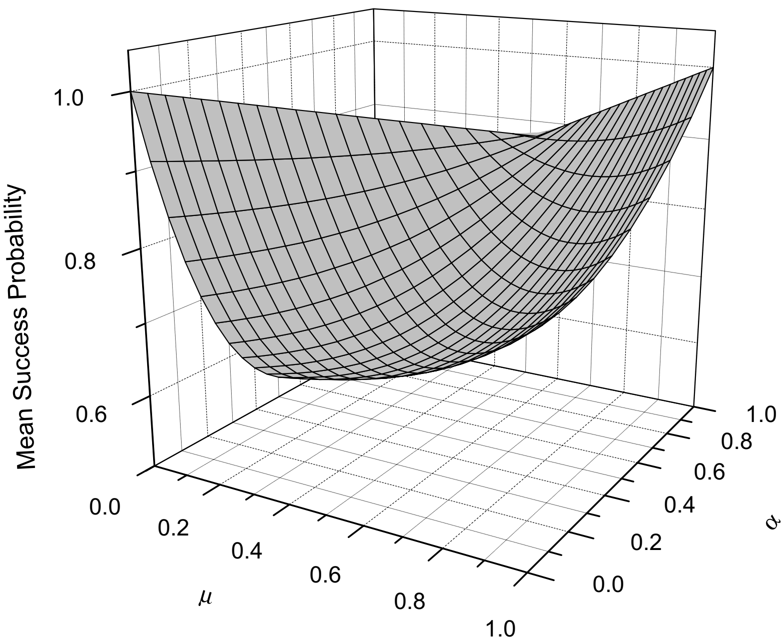

FIG. 6: Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of memory parameter μ and quantum noise parameter α for depolarizing channel.

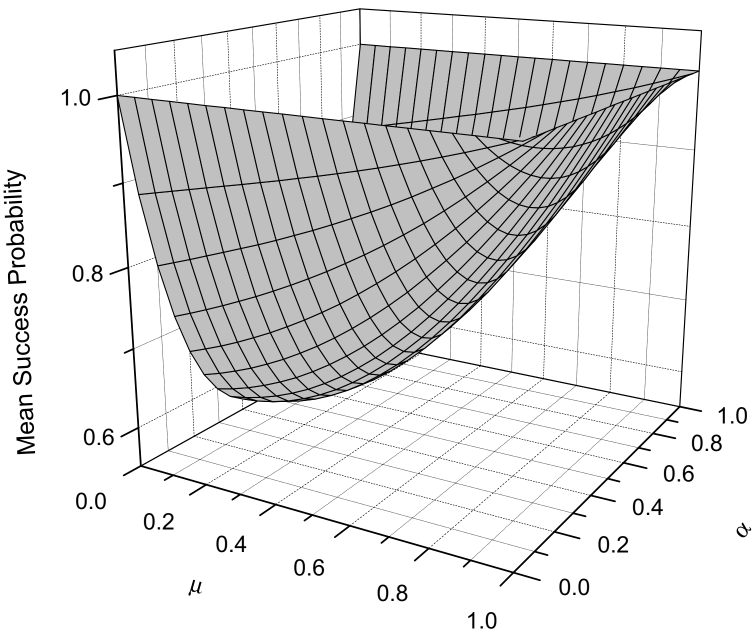

FIG. 7: Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of memory parameter μ and quantum noise parameter α for phase flip channel.

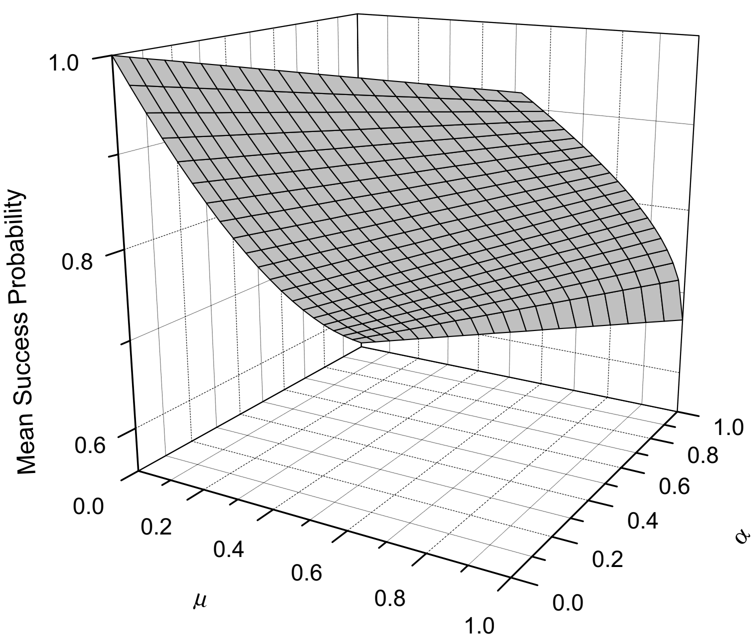

FIG. 8: Mean success probability $\bar{P}(\alpha, \mu)$ plotted as a function of memory parameter μ and quantum noise parameter α for phase damping channel.

TABLE I: Single qubit Kraus operators for typical noise channels such as depolarizing, amplitude damping, phase damping, phase flip, bit flip and bit-phase flip where α represents the quantum noise parameter.

Depolarizing channel	$E_0 = \sqrt{1 - \frac{3\alpha}{4}} I, \quad E_1 = \sqrt{\frac{\alpha}{4}}\sigma_x$ $E_2 = \sqrt{\frac{\alpha}{4}}\sigma_y, \quad E_3 = \sqrt{\frac{\alpha}{4}}\sigma_z$
Amplitude damping channel	$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\alpha} \end{bmatrix}, E_1 = \begin{bmatrix} 0 & \sqrt{\alpha} \\ 0 & 0 \end{bmatrix}$
Phase damping channel	$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\alpha} \end{bmatrix}, E_1 = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{\alpha} \end{bmatrix}$
Phase flip channel	$E_0 = \sqrt{1-\alpha} I, \quad E_1 = \sqrt{\alpha}\sigma_z$
Bit flip channel	$E_0 = \sqrt{1-\alpha} I, \quad E_1 = \sqrt{\alpha}\sigma_x$
Bit-phase flip channel	$E_0 = \sqrt{1-\alpha} I, \quad E_1 = \sqrt{\alpha}\sigma_y$

TABLE II: Success probability $P(\alpha,\mu)$ for all combinations of magic squares game inputs for depolarizing, amplitude damping, phase damping, phase flip, bit flip and bit-phase flip channels in the presence of memory.

Depolarizing channel:	
$\{(i,j) \mid i,j = 1,2,3\}$	$P_{ij}(\alpha,\mu) = \frac{1}{2}(2 - 4\alpha + 3\mu\alpha + \mu^3\alpha + 6\alpha^2 - 9\mu\alpha^2$ $+3\mu^2\alpha^2 - 4\alpha^3 + 9\mu\alpha^3 - 6\mu^2\alpha^3 + \mu^3\alpha^3$ $+\alpha^4 - 3\mu\alpha^4 + 3\mu^2\alpha^4 - \mu^3\alpha^4)$
Amplitude damping channel:	
$i,j \in \{(1,1),(1,2),(2,3),(3,3)\}$	$P_{ij}(\alpha,\mu) = \frac{1}{4}(4 - 4\alpha + 3\mu\alpha + 2\alpha^2 - 2\mu\alpha^2)$
$i,j \in \ (1,3)$	$P_{ij}(\alpha,\mu) = 1 - 2\alpha + 2\mu\alpha + 2\alpha^2 - 2\mu\alpha^2$
$i,j \in \{(2,1),(2,2),(3,1),(3,2)\}$	$P_{ij}(\alpha,\mu) = \frac{1}{2}(2 - \mu + \mu\sqrt{1-\alpha} - 3\alpha + 3\mu\alpha + 2\alpha^2 - 2\mu\alpha^2)$
Phase damping channel:	
$i,j \in \{(1,1),(1,2),(2,3),(3,3)\}$	$P_{ij}(\alpha,\mu) = \frac{1}{4}(4 - 4\alpha + 3\mu\alpha + 2\alpha^2 - 2\mu\alpha^2)$
$i,j \in \ (1,3)$	$P_{ij}(\alpha,\mu) = 1$
$i,j \in \{(2,1),(2,2),(3,1),(3,2)\}$	$P_{ij}(\alpha,\mu) = \frac{1}{2}(2 - \mu + \mu\sqrt{1-\alpha} - \alpha + \mu\alpha)$
Phase Flip channel:	
$i,j \in \{(1,1),(1,2),(2,3),(3,3)\}$	$P_{ij}(\alpha,\mu) = 1 - 4\alpha + 6\mu\alpha - 4\mu^2\alpha + 2\mu^3\alpha + 12\alpha^2 + 24\mu^2\alpha^4$ $-30\mu\alpha^2 + 28\mu^2\alpha^2 - 10\mu^3\alpha^2 - 16\alpha^3 + 48\mu\alpha^3$ $-48\mu^2\alpha^3 + 16\mu^3\alpha^3 + 8\alpha^4 - 24\mu\alpha^4 - 8\mu^3\alpha^4$
$i,j \in \ (1,3)$	$P_{ij}(\alpha,\mu) = 1$
$i,j \in \{(2,1),(2,2),(3,1),(3,2)\}$	$P_{ij}(\alpha,\mu) = 1 - 2\alpha + 2\mu^2\alpha + 2\alpha^2 - 2\mu^2\alpha^2$
Bit flip channel:	
$i,j \in \{(1,1),(1,2),(3,1),(3,2)\}$	$P_{ij}(\alpha,\mu) = 1 - 2\alpha + 2\mu^2\alpha + 2\alpha^2 - 2\mu^2\alpha^2$
$i,j \in \ (2,3)$	$P_{ij}(\alpha,\mu) = 1$
$i,j \in \{(1,3),(2,1),(2,2),(3,3)\}$	$P_{ij}(\alpha,\mu) = 1 - 4\alpha + 6\mu\alpha - 4\mu^2\alpha + 2\mu^3\alpha + 12\alpha^2 + 24\mu^2\alpha^4$ $-30\mu\alpha^2 + 28\mu^2\alpha^2 - 10\mu^3\alpha^2 - 16\alpha^3 + 48\mu\alpha^3$ $-48\mu^2\alpha^3 + 16\mu^3\alpha^3 + 8\alpha^4 - 24\mu\alpha^4 - 8\mu^3\alpha^4$
Bit-phase flip channel:	
$i,j \in \{(1,1),(1,2),(2,1),(2,2)\}$	$P_{ij}(\alpha,\mu) = 1 - 2\alpha + 2\mu^2\alpha + 2\alpha^2 - 2\mu^2\alpha^2$
$i,j \in \ (3,3)$	$P_{ij}(\alpha,\mu) = 1$
$i,j \in \{(1,3),(2,3),(3,1),(3,2)\}$	$P_{ij}(\alpha,\mu) = 1 - 4\alpha + 6\mu\alpha - 4\mu^2\alpha + 2\mu^3\alpha + 12\alpha^2 + 24\mu^2\alpha^4$ $-30\mu\alpha^2 + 28\mu^2\alpha^2 - 10\mu^3\alpha^2 - 16\alpha^3 + 48\mu\alpha^3$ $-48\mu^2\alpha^3 + 16\mu^3\alpha^3 + 8\alpha^4 - 24\mu\alpha^4 - 8\mu^3\alpha^4$

TABLE III: Mean success probability $\bar{P}(\alpha,\mu)$ for depolarizing, amplitude damping, phase damping and flipping (phase flip, bit flip or bit-phase flip) channels in the presence of memory.

Depolarizing channel:
$\bar{P}(\alpha,\mu) = \frac{1}{2}(2-4\alpha+3\mu\alpha+\mu^3\alpha+6\alpha^2-9\mu\alpha^2+3\mu^2\alpha^2-4\alpha^3 +9\mu\alpha^3-6\mu^2\alpha^3+\mu^3\alpha^3+\alpha^4-3\mu\alpha^4+3\mu^2\alpha^4-\mu^3\alpha^4)$
Amplitude damping channel:
$\bar{P}(\alpha,\mu) = 1/9(9-2\mu+2\mu\sqrt{1-\alpha}-12\alpha+11\mu\alpha+8\alpha^2-8\mu\alpha^2)$
Phase damping channel:
$\bar{P}(\alpha,\mu) = 1/9(9-2\mu+2\mu\sqrt{1-\alpha}-6\alpha+5\mu\alpha+2\alpha^2-2\mu\alpha^2)$
Flipping channels:
$\bar{P}(\alpha,\mu) = 1/9(9-24\alpha+24\mu\alpha-8\mu^2\alpha+8\mu^3\alpha+56\alpha^2-120\mu\alpha^2 +104\mu^2\alpha^2-40\mu^3\alpha^2-64\alpha^3+192\mu\alpha^3-192\mu^2\alpha^3 +64\mu^3\alpha^3+32\alpha^4-96\mu\alpha^4+96\mu^2\alpha^4-32\mu^3\alpha^4)$

Table 1: Single qubit Kraus operators for typical noise channels such as depolarizing, amplitude damping, phase damping, phase flip, bit flip and bit-phase flip.

Depolarizing channel	$E_0 = \sqrt{1-\frac{3\alpha}{4}}I, \quad E_1 = \sqrt{\frac{\alpha}{4}}\sigma_x$ $E_2 = \sqrt{\frac{\alpha}{4}}\sigma_y, \quad E_3 = \sqrt{\frac{\alpha}{4}}\sigma_z$
Amplitude damping channel	$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\alpha} \end{bmatrix}, E_1 = \begin{bmatrix} 0 & \sqrt{\alpha} \\ 0 & 0 \end{bmatrix}$
Phase damping channel	$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\alpha} \end{bmatrix}, E_1 = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{\alpha} \end{bmatrix}$
Phase flip channel	$E_0 = \sqrt{1-\alpha}I, \quad E_1 = \sqrt{\alpha}\sigma_z$
Bit flip channel	$E_0 = \sqrt{1-\alpha}I, \quad E_1 = \sqrt{\alpha}\sigma_x$
Bit-phase flip channel	$E_0 = \sqrt{1-\alpha}I, \quad E_1 = \sqrt{\alpha}\sigma_y$

Table 1: Mean success probability for depolarizing, amplitude damping, phase damping and flipping channels with memory.

	Depolarizing channel:
$\bar{P}(\alpha,\mu)=$	$\frac{1}{2}(2-4\alpha+3\mu\alpha+\mu^3\alpha+6\alpha^2-9\mu\alpha^2+3\mu^2\alpha^2-4\alpha^3$ $+9\mu\alpha^3-6\mu^2\alpha^3+\mu^3\alpha^3+\alpha^4-3\mu\alpha^4+3\mu^2\alpha^4-\mu^3\alpha^4)$
	Amplitude damping channel:
$\bar{P}(\alpha,\mu)=$	$1/9(9-2\mu+2\mu\sqrt{1-\alpha}-12\alpha+11\mu\alpha+8\alpha^2-8\mu\alpha^2)$
	Phase damping channel:
$\bar{P}(\alpha,\mu)=$	$1/9(9-2\mu+2\mu\sqrt{1-\alpha}-6\alpha+5\mu\alpha+2\alpha^2-2\mu\alpha^2)$
	Flipping channels:
$\bar{P}(\alpha,\mu)=$	$1/9(9-24\alpha+24\mu\alpha-8\mu^2\alpha+8\mu^3\alpha+56\alpha^2-120\mu\alpha^2$ $+104\mu^2\alpha^2-40\mu^3\alpha^2-64\alpha^3+192\mu\alpha^3-192\mu^2\alpha^3$ $+64\mu^3\alpha^3+32\alpha^4-96\mu\alpha^4+96\mu^2\alpha^4-32\mu^3\alpha^4)$

Table 2: Mean success probability for depolarizing, amplitude damping, phase damping and flipping channels with memory.

	Depolarizing channel:
$\bar{P}(\alpha,\mu)=$	$\frac{1}{2}(2-4\alpha+3\mu\alpha+\mu^3\alpha+6\alpha^2-9\mu\alpha^2+3\mu^2\alpha^2-4\alpha^3$ $+9\mu\alpha^3-6\mu^2\alpha^3+\mu^3\alpha^3+\alpha^4-3\mu\alpha^4+3\mu^2\alpha^4-\mu^3\alpha^4)$
	Amplitude damping channel:
$\bar{P}(\alpha,\mu)=$	$1/9(9-2\mu+2\mu\sqrt{1-\alpha}-12\alpha+11\mu\alpha+8\alpha^2-8\mu\alpha^2)$
	Phase damping channel:
$\bar{P}(\alpha,\mu)=$	$1/9(9-2\mu+2\mu\sqrt{1-\alpha}-6\alpha+5\mu\alpha+2\alpha^2-2\mu\alpha^2)$
	Flipping channels:
$\bar{P}(\alpha,\mu)=$	$1/9(9-24\alpha+24\mu\alpha-8\mu^2\alpha+8\mu^3\alpha+56\alpha^2-120\mu\alpha^2$ $+104\mu^2\alpha^2-40\mu^3\alpha^2-64\alpha^3+192\mu\alpha^3-192\mu^2\alpha^3$ $+64\mu^3\alpha^3+32\alpha^4-96\mu\alpha^4+96\mu^2\alpha^4-32\mu^3\alpha^4)$